

DECIMAL NUMBERS AND SIGNIFICANT ZEROS AND . . . MONEY!

Zeros are important! Even though a zero on its own has no value (after all, 'zero' means 'nothing'), it can affect the value of numbers containing it. When numbers are written in decimal form you will often see zeros included. But which ones are significant and which ones can be ignored? The best way to work it out is to ask yourself this question:

If I remove this zero, will it affect the value of the number? If it does affect the value, then that particular zero is **significant** and cannot be ignored. If removal of the zero does not affect the value of the number then it is **not significant** and can be ignored.

If some zeros are not significant, why do you sometimes see them written in numbers? Often it is because the number has been generated by a computer and must contain a set number of **digits** (individual numbers). Here's a list of computer-generated numbers:

83427 05623 90090 00002 02029

The **values** of these numbers are:

83427 5623 90090 2 2029

Some numbers include a decimal point, and digits to the right of it. Here's another list of computer-generated numbers:

83427•628 05623•500 90090•506 00002•003 02029•770

The **values** of these numbers are:

83427•628 5623•5 90090•506 2•003 2029•77

Let's write down the computer-generated list again and this time highlight the non-significant zeros in grey ink:

83427•628 05623•500 90090•506 00002•003 02029•770

You can see that the grey zeros do not affect the value of the numbers they feature in and so they are **not significant**. They are the **first** zeros of the part of the number to the **left** of the decimal point and the **last** zeros of the part of the number to the **right** of the decimal point. The other zeros would affect the value of the numbers if they were removed, so they are **significant**.

Let's remind ourselves what the decimal point actually means. All numbers to the **left** of the decimal point have values **greater** than (or equal to) 1. All numbers to the **right** of the decimal point have values **less** than 1. Let's look at our first number again in more detail:

Ten thousands	Thousands	Hundreds	Tens	Units	Dec point	Tenths	Hundredths	Thousandths	Ten thousandths
10,000	1000	100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$
8	3	4	2	7	.	6	2	8	

What does this mean? We can say that the number **83427.628** is made up of:

8 ten thousands = 80,000
 3 thousands = 3,000
 4 hundreds = 400
 2 tens = 20
 7 units = 7

6 tenths = •6 (6 divided by 10)
 2 hundredths = •02 (2 divided by 100)
 8 thousandths = •008 (8 divided by 1000)

MULTIPLICATION

Remember: the **decimal** system is based on the number **10** (Latin name **decem**). When we want to multiply a number by **10**, we write a zero at the end of the number. If we want to multiply by **100** we write two zeros, and so on.

Example: $83427 \times 10 = 834270$

Example: $83427 \times 100 = 8342700$

If the number includes a decimal point then to multiply by 10 we move the decimal point **one** digit to the **right**.

8 3 4 2 7 • 6 2 8

Example: $83427.628 \times 10 = 834276.28$

With large numbers, it is usual to separate the number into blocks of three digits, starting at the decimal point and moving left. This makes it easier to read. So we would write, for example, **834,276.28**. This number is eight hundred and thirty-four thousand, two hundred and seventy-six point two eight.

If we want to multiply a number by **100** we move the decimal point **two** digits to the **right**.

8 3 4 2 7 • 6 2 8

Example: $83427.628 \times 100 = 8342762.8$

Again, we'll make it easier to read by separating the number into blocks of three digits. So **8342762.8** would be written **8,342,762.8**. This number is eight million, three hundred and forty-two thousand, seven hundred and sixty-two point eight.

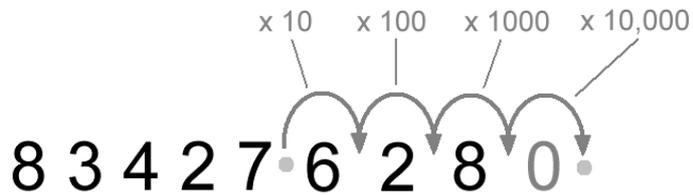
If we want to multiply a number by **1000** we move the decimal point **three** digits to the **right**. If we want to multiply by **10,000** we move the decimal point **four** digits to the **right** and so on.

Example: $83427.628 \times 1000 = 83427628.$

Interesting! We've got nothing after the decimal point! What does that mean? If the decimal point ends up just to the right of the last digit in the number, as in this example, you can remove it completely. So **83427628.** can be written as **83427628**, or more clearly, **83,427,628**.

Example: $83427.628 \times 10,000 = 83427628????$

We've run out of digits for the decimal point to jump over! What should we do? Luckily, there is an answer: write zeros in the spaces the decimal point would jump over and then remove the decimal point.

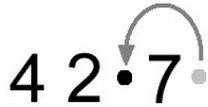


So: $83427.628 \times 10,000 = 834276280.$ or $834,276,280$

This number is eight hundred and thirty-four million, two hundred and seventy-six thousand, two hundred and eighty.

DIVISION

Let's go on to division. You'll remember that to **multiply** by 10 we move the decimal point one digit to the **right**. Not surprisingly, if we want to **divide** by 10, it's just the opposite – we move the decimal point to the **left**. If there is no decimal point, imagine that it sits just to the right of the last digit in the number.

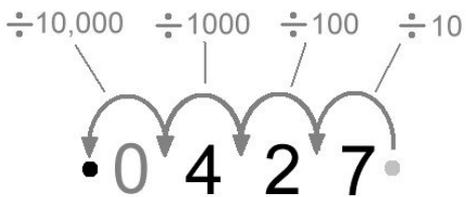
$$427. \quad 42.7$$


Example: $427 \div 10 = 42.7$

To divide by **100**, we move the decimal point **two** digits to the left, and so on. If we divide **427** by **1000** we will get the answer **•427**. To make it clearer we would write a zero to the left of the decimal point, even though it's not a significant zero, because removing it does not affect the value of the number.

So: $427 \div 1000 = \bullet 427$ or $0\bullet 427$

Suppose we want to divide 427 by 10,000. We run out of digits for the decimal point to jump over! As with multiplication, we solve the problem by writing zeros in the spaces the decimal point would jump over.

$$\begin{array}{cccc} \div 10,000 & \div 1,000 & \div 100 & \div 10 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \bullet 0 & 4 & 2 & 7 \end{array}$$


Again, we can put a zero to the left of the decimal point to make the number clearer. So we would write **0•427**. But remember, this zero is **not** a significant zero, because removing it does not affect the value of the number.

What does the number **0.0427** mean?

0	ten thousands	=	00,000
0	thousands	=	0,000
0	hundreds	=	000
0	tens	=	00
0	units	=	0
0	tenths	=	•0 (0 divided by 10)
4	hundredths	=	•04 (4 divided by 100)
2	thousandths	=	•002 (2 divided by 1000)
7	ten thousandths	=	•0007 (7 divided by 10,000)

Wow! It all looks very complicated, doesn't it! But re-read the lesson a couple of times and you will see that we're just applying basic logic to the decimal system, which itself is not too difficult to understand.

MONEY

There are many practical examples of decimal numbers, one of the most important being money. We know that there are one hundred pence (pennies) in one pound. So one penny is one pound divided by 100. How do we write that down? We'll imagine a decimal point to the right of our number 1 and then move it two places to the left, writing in a zero where there is a space to jump over:

$$\begin{array}{ccccccc}
 1\cdot & \cdot 0 & 1\cdot & = & \cdot 01 & \pounds 0\cdot 01 \\
 \pounds 1 & & \pounds 1 & = & 1\text{p} & 1\text{p} \\
 & & \frac{\quad}{100} & & &
 \end{array}$$

So 1p is $\pounds 0\cdot 01$. 10p is obviously 1p times 10. We know how to do that:

$$\cdot 01 \quad \cdot 0\cdot 1 \quad 0\cdot 1 \quad 0\cdot 10$$

So 10p is $\pounds 0\cdot 10$ and 50p is $\pounds 0\cdot 50$. Although the last zeros are not significant (because if you remove them it does not affect the value of the money) it's usual to leave them in place so that you can clearly see how many pence are there.

If we found we had, say, three twenty pound notes, one ten pound note, two five pound notes, four pound coins, three fifty pence coins, twelve ten pence coins and four pennies, how much would we have?

3	x	20	=	60	
1	x	10	=	10	
2	x	5	=	10	
4	x	1	=	4	
3	x	0·50	=	1·50	(because 150p = £1 and 50p)
12	x	0·10	=	1·20	(because 120p = £1 and 20p)
4	x	0·01	=	0·04	
Total				86·74	

We would have a total of £86 and 74p.

See! Decimals are important!